



RATIOS

CONCEPT BUILDER MANUAL

Ratios

Ratios:

A ratio is simply a fraction. The following notations all express the ratio of x to y
 $= x : y, x \div y, \text{ or } x/y.$

In the ratio $x : y$, we call a as the first term or antecedent and b , the second term or consequent.

Writing two numbers as a ratio provides a convenient way to compare their sizes. For example, since $3/\pi < 1$, we know that 3 is less than π .

A ratio compares two numbers. Just as you cannot compare apples and oranges, so the numbers you are comparing must have the same units.

For example, you cannot form the ratio of 2 feet to 4 meters because the two numbers are expressed in different units - feet vs. meters.

Example:

1. What is the ratio of 2 feet to 4 yards?

(A) 1 : 2 (B) 1 : 8 (C) 1 : 7 (D) 1 : 6 (E) 1 : 5

Solution:

The ratio cannot be formed until the numbers are expressed in the same units. Let's turn the yards into feet.

Since there are 3 feet in a yard, 4 yards = 4×3 feet = 12 feet .

Forming the ratio yields 2 feet / 12 feet = $1/6$ or 1:6

The answer is (D).

Note:

Taking the reciprocal of a fraction usually changes its size.

For example: $3/4 \neq 4/3$

So order is important in a ratio = $3:4 \neq 4:3$.

Rule:

The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Example:

$4 : 5 = 8 : 10 = 12 : 15$

Also, $4 : 6 = 2 : 3$

PROPORTION:

The equality of two ratios (fractions) is called proportion. If $a : b = c : d$, we write $a : b :: c : d$ and we say that a, b, c, d are in proportion.

Here a and d are called *extremes*, while b and c are called *mean* terms.

Product of means = Product of extremes, thus,

$$a : b :: c : d = (b \times c) = (a \times d)$$

Types:

1. Fourth Proportional

If $a : b = c : d$, then d is called the fourth proportional to a, b, c

2. Third Proportional

$a : b = c : d$, then c is called the third proportional to a and b

3. Mean Proportional

Mean proportional between a and b is ab

4. Compounded Ratio:

The compounded ratio of the ratios: $(a : b), (c : d), (e : f)$ is $(ace : bdf)$

Variation

We say that x is directly proportional to y , if $x = ky$ for some constant k and we write, $x \propto y$

We say that x is inversely proportional to y , if $xy=k$ for some constant k and we write, $x \propto 1/y$

Sample Questions:

1. A sum of money is to be distributed among A, B, C, D in the proportion of 5:2:4:3. If C gets Rs.1000 more than D, what is B's share?
- 500
 - 1500
 - 2000
 - None

Solution:

Let the shares of A, B, C and D be Rs. $5x$, Rs. $2x$, Rs. $4x$ and Rs. $3x$ respectively.

Then, $4x - 3x = 1000 \Rightarrow x = 1000$.

B's share = Rs. $2x = \text{Rs.}(2 \times 1000)$

= Rs.2000

2. A sum of Rs.312 was divided among 100 boys and girls in such a way that the boy gets Rs.3.60 and each girl Rs.2.40 the number of girls is?
- 35
 - 40
 - 45
 - 50

Solution:

Step (i) Let x be the number of boys and y be the number of girls.

Given total number of boys and girls = 100, $x + y = 100$

Step (ii) A boy gets Rs.3.60 and a girl gets Rs.2.40

The amount given to 100 boys and girls = Rs. 312

$3.6x + 2.4y = 312$

Step (iii) Solving (i) and (ii)

$$3.6x + 3.6y = 360 \text{ multiplying by } 3.6$$

$$3.6x + 2.4y = 312$$

$$1.20y = 48$$

$$y = 40$$

The number of girls is 40.

3. If 20 men or 24 women or 40 boys can do a job in 12 days working for 8 hours a day, how many men working with 6 women and 2 boys take to do a job four times as big working for 5 hours a day for 12 days?
- 120 men
 - 122 men
 - 128 men
 - 134 men

Solution:

Let's try solving this Problem using ratio approach.

Amount of work done by 20 men = 24 women = 40 boys or 1 man = 1.2 woman = 2 boys.
Let us, therefore, find out the amount of men required, if only men were working on the job, to complete the new job under the new conditions and then make adjustments for the women and children working with the men.

The man hours required to complete the new job = 4 times the man hours required to complete the old job. (As the new job is 4 times as big as the old job)

Let n be the number of men required.

$$n \times 5 \times 12 = 20 \times 8 \times 12 \times 4 \text{ or } n = 128$$

i.e. 128 men working on the job will be able to complete the given job.

However, the problem states that 6 women and 2 boys are working on the job.

6 women = 5 men and 2 boys = 1 man.

\therefore The equivalent of $5+1 = 6$ men are already working.

Thus, final number of men working, = $128 - 6 = 122$ men

4. P, Q and R enter into a partnership with capitals in the ratio 3:2:1. After 4 months, P leaves the business and after 4 more months Q also leaves the business and R continues till the end of the year. If R takes 10 of the profit for managing the business, then what part of the profit does R get?
- 37%
 - 36%
 - 27%
 - 30%

Solution:

Let Rs 100 be the profit.

Rs 90 is to be divided in the ratio 12 : 16 : 12. i.e 3 : 4 : 3

R gets $\frac{3}{10} \times 90 = 27$ and 10 for managing
Thus $27+10=37\%$

5. A fort has provisions for 60 days. If after 15 days 500 men strengthen them and the food lasts 40 days longer, how many men are there in the fort?
- 3500
 - 4000
 - 6000
 - None

Solution:

Let there be 'x' men in the beginning so that after 15 days the food for them is left for 45 days.

After adding 500 men the food lasts for only 40 days.

Now $(x+500)$ men can have the same food for 40 days.

Therefore, by equating the amount of food we get,

$$45x = (x+500) \times 40$$

$$5x = 20,000$$

$$x=4000$$

Therefore, there are 4,000 men in the fort.

