BANK NAUKRI

BANK NAUKRI

NUMBER SYSTEM

CONCEPT BUILDER MANUAL

NUMBER SYSTEM:

BASIC FORMULAE:

1. $(a+b)^2 = a^2 + b^2 + 2ab$

2. $(a-b)^2 = a^2 + b^2 - 2ab$

3. $(a+b)^2 - (a-b)^2 = 4ab$

4. $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

5. $(a^2 - b^2) = (a+b)(a-b)$

6. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$

7. $(a^3 + b^3) = (a+b)(a^2-ab+b^2)$

8. $(a^3 - b^3) = (a-b)(a^2+ab+b^2)$

9. $(a^3+b^3+c^3-3abc) = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

10. If $a+b+c = 0$, then $a^3+b^3+c^3 = 3abc$

TYPES OF NUMBERS:

NATURAL NUMBERS:

1. Counting numbers 1, 2, 3, 4, 5, ... are called natural numbers

WHOLE NUMBERS:

1. All counting numbers together with zero form the set of whole numbers.Thus,

   (i) 0 is the only whole number which is not a natural number.
   (ii) Every natural number is a whole number.

INTERGERS:

1. All natural numbers, 0 and negatives of counting numbers i.e., ...,−3,−2,−1,0,1,2,3,..... together form the set of integers.

   (i) *Positive Integers*: 1,2,3,4, ... is the set of all positive integers.
   (ii) *Negative Integers*: −1,−2,−3, ... is the set of all negative integers.

(iii) *Non-Positive and Non-Negative Integers*: 0 is neither positive nor negative. So, 0,1,2,3, ... represents the set of non-negative integers, while 0,−1,−2,−3, ... represents the set of non-positive integers.

EVEN NUMBERS:

1. A number divisible by 2 is called an even number, e.g., 2, 4, 6, 8…

ODD NUMBERS:

1. A number not divisible by 2 is called an odd number. e.g.,1,3,5,7,9,11,

PRIME NUMBERS:

1. A number greater than 1 is called a prime number, if it has exactly two factors, namely 1 and the number itself.
2. 2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97 are the prime numbers within 100

COMPOSITE NUMBER

1. Numbers greater than 1 which are not prime, are known as composite numbers, e.g., 4, 6, 8, 9, 10, 12.

   Note:
   (i) 1 is neither prime nor composite.
   (ii) 2 is the only even number which is prime.
   (iii) There are 25 prime numbers between 1 and 100.

EVEN AND ODD NUMBER:

1. A number n is even if the remainder is zero when n is divided by 2 : n = 2z + 0, or n = 2z
2. A number n is odd if the remainder is one when n is divided by 2 : n = 2z + 1.
3. The following properties for odd and even numbers are very useful
   - even X even = even
   - odd X odd = odd
   - even X odd = even
   - even + even = even
   - odd + odd = even
   - even + odd =odd

TESTS OF DIVISIBILITY:

DIVISIBILITY BY 2:

1. A number is divisible by 2 if its unit's digit is any of 0,2,4,6,8.
   Example: 84932 is divisible by 2, while 65935 is not.

DIVISIBILITY BY 3:

1. A number is divisible by 3 if the sum of its digits is divisible by 3.
   Example: 592482 is divisible by 3, since sum of its digits = (5+9+2+4+8+2) = 30, which is divisible by 3. But, 864329 is not divisible by 3, since sum of its digits = (8+6+4+3+2+9) = 32, which is not divisible by 3.

DIVISIBILITY BY 4:

1. A number is divisible by 4 if the number formed by the last two digits is divisible by 4.
   Example: 892648 is divisible by 4 since the number formed by the last two digits is 48, which is divisible by 4. But, 749282 is not divisible by 4, since the number formed by the last two digits is 82, which is not divisible by 4.

DIVISIBILITY BY 5:

1. A number is divisible by 5 if its unit's digit is either 0 or 5. Thus, 20820 and 50345 are divisible by 5, while 30934 and 40946 are not.

DIVISIBILITY BY 6:

1. A number is divisible by 6 if it is divisible by both 2 and 3.
   Example: The number 35256 is clearly divisible by 2. Sum of its digits = (3+5+2+5+6) = 21, which is divisible by 3. Thus, 35256 is divisible by 2 as well as 3. Hence, 35256 is divisible by 6.

DIVISIBILITY BY 8:

1. A number is divisible by 8 if the number formed by the last Three digits of the given number is divisible by 8.
   Example: 953360 is divisible by 8 since the number formed by last three digits is 360, which is divisible by 8. But, 529418 is not divisible by 8, since the number formed by last three digits is 418, which is not divisible by 8.

DIVISIBILITY BY 9:

1. A number is divisible by 9 if the sum of its digits is divisible by 9.

Example: 60732 is divisible by 9, since sum of digits = (6+0+7+3+2) = 18, which is divisible by 9. But, 68956 is not divisible by 9, since sum of digits = (6+8+9+5+6) = 34, which is not divisible by 9.

DIVISIBILITY BY 10:

1. A number is divisible by 10 if it ends with 0.
   Example: 96410, 10480 are divisible by 10, while 96375 is not.

PROGRESSION:

A succession of numbers formed and arranged in a definite order according to a certain definite rule is called a progression.

Arithmetic Progression (A.P.):

If each term of a progression differs from its preceding term by a constant, then such a progression is called an arithmetical progression. This constant difference is called the common difference of the A.P.
An A.P. with first term a and common difference d is given by a, (a+d), (a+2d), (a+3d), ...
The nth term of this A.P. is given by $T_n = a+(n-1)d$.
The sum of n terms of this A.P. $S_n = (n / 2) X [2a+(n-1)d]$
$=(n/2) X$ (first term + last term).

Results

(i) $(1+2+3+....+n) = n(n+1) / 2$
(ii) $(1^2+2^2+3^2+...+n^2) = n(n+1)(2n+1) / 6$
(iii) $(1^3+2^3+3^3+...+n^3) = n^2(n+1)^2 /4$

Geometrical Progression (G.P.):

A progression of numbers in which every term bears a constant ratio with its preceding term, is called a geometrical progression. The constant ratio is called the common ratio of the G.P.
A G.P. with first term a and common ratio r is : a, ar, ar^2, ...

In this G.P.nth term, $T_n = ar^{(n-1)}$

sum of n terms, $S_n = a(1-r^n) / (1-r)$ when r>1

SAMPLE QUESTIONS:

1. What is the value of M and N respectively if M39048458N is divisible by 8 and 11, where M and N are single digit integers?
   a. 7,8

b. 8,6
c. 6,4
d. 5,4

Solution:
A number is divisible by 8, if the number formed by the last three digits is divisible by 8.
i.e 58N is divisible by 8 = N=4
Again a number is divisible by 11, if the difference between the sum of digits at even places and sum of digits at the odd places is either 0 or divisible by 11.
i.e, (M+9+4+4+8) − (3+0+8+5+N) = M−N+9 = M+5
It cannot be zero hence, M+5=11
M=6.

2. If n is a positive integer, which one of the following numbers must have a remainder of 3 when divided by any of the numbers 4, 5 and 6?
   a. 12n + 3
   b. 24n + 3
   c. 90n + 2
   d. 120n + 3

Solution:

Let m be a number that has a remainder of 3 when divided by any of the numbers 4, 5 and 6. Then m−3 must be exactly divisible by all three numbers. Hence, m−3 must be a multiple of the Least Common Multiple of the numbers 4, 5 and 6.
The LCM is 3×4×5=60. Hence, we can suppose m−3=60p, where p is a positive integer.
Replacing p with n, we get m–3 = 60n. So, m = 60n + 3.
Therefore, D is in the same format 120n + 3 = 60(2n) + 3

3. When writing numbers from 1 to 10,000, how many times is the digit 9 written?
   a. 3200
   b. 3600
   c. 4000
   d. 4200

Solution:

The digits 9 occurs in the thousands place in 1000 numbers. It occurs in the hundreds place in 1000 numbers and so on…The digit occurs 4000 times.

4. How many keystrokes are needed to type numbers from 1 to 1000 on a standard keyboard?
   a. 3001
   b. 2893

c. 2704
d. 2890

Solution:

While typing numbers from 1 to 1000, you have 9 single digit numbers from 1 to 9. Each of them requires one keystroke. That is 9 key strokes.
There are 90 two-digit numbers, from 10 to 99. Each of these numbers requires 2 keystrokes. Therefore, one requires 180 keystrokes to type the 2 digit numbers.
There are 900 three-digit numbers, from 100 to 999. Each of these numbers requires 3 keystrokes. Therefore, one requires 2700 keystrokes to type these 3 digit numbers.
Then 1000 is a four-digit number which requires 4 keystrokes.
Totally, therefore, one requires 9+180+2700+4 = 2893 keystrokes.

5. Let n be the number of different 5 digit numbers, divisible by 4 with the digits 1, 2, 3, 4, 5 and 6, no digit being repeated in the numbers. What is the value of n?
   a. 144
   b. 168
   c. 192
   d. None of these

   Solution:
   Test of divisibility by 4 is that the last two digits should be divisible by 4.
   Case 1: When the last 2 digits are 12, i.e., _ _ _12=4×3×2=24 numbers
   Case 2: When the last 2 digits are 16, there are 24 numbers
   Case 3: When the last 2 digits are 24 there are 24 numbers
   Case 4: When the last 2 digits are 32 there are 24numbers
   Case 5: When last 2 digits are 36 there are 24 numbers
   Case 6: When last 2 digits are 52 there are 24 numbers
   Case 7: When last 2 digits are 56 there are 24 numbers
   Case 8: When last 2 digits are 64 there are 24 numbers
   Total = 8 X 24 = 192